

Nonparametric Estimation of Distribution Function for Stratified Populations

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Abstract Nonparametric estimation of population parameters for finite populations has been used with great success for data that fit the independent and identically distributed framework. However, most of these approaches do not extend to data from multistage samples. In this work, we present a method for developing a nonparametric distribution function for a finite population that has been stratified. Proportional allocation of sampling weights has been utilized alongside kernel weights. Asymptotic properties of the estimator are derived and are compared with those of existing model based estimators using the simulated sets of data. The results show that applying the bias reduction technique to a stratified population greatly improves precision of the estimator.

Keywords Stratified Sampling, Proportional Allocation, α -Quantile, Multiplicative Bias Correction

1. Introduction

Estimation of population parameters is a fundamental issue in statistics because such quantities are necessary components in most theoretical studies and practical applications. The main idea of nonparametric statistics is to make inferences about unknown quantities without resorting to parametric reduction of the problem. Example, suppose that a random variable X has a distribution function F . The approach taken by parametric statistics is to assume that F belongs to a family of distributions that can be explained by a smaller number of parameters. These parameters are then estimated and inference is made about the quantities of interest.

Clearly, the parametric approach relies on a tremendous reduction of the original problem. It assumes that all uncertainty regarding the distribution function can be reduced to just one or two unknown numbers. If these assumptions are true, then there is nothing wrong in making the assumptions. However if they are false, the resulting inference will be questionable and we might miss the interesting patterns in the data.

On the contrary, nonparametric statistics tries to make as few assumptions as possible about the data. For instance, it allows F to be any function provided it satisfies the definition

of a distribution function. This requires the development of a whole new set of tools and instead of estimating parameters, the nonparametric approach estimates the function.

A number of estimation procedures have been developed to estimate the distribution of a random variable in the past (Zhao et al., 2013). For more insight on this see (Chambers and Dunstan, 1986), (Kuk, 1993), (Rao et al 1990) and (Dorfman & Hall, 1993). (Breunig, 2008) also considered a weighted, nonparametric density estimator for stratified samples. He derived the optimal bandwidth and provided a plug-in bandwidth when all strata are normally distributed. (Chambers and Clark, 2012) gave a general nonparametric methodology for estimating a distribution function for a stratified population using a linear regression model.

Despite the success of using the nonparametric approach in the estimation of population parameters, there exists some tendency of the estimators being biased. Moreover, kernel smoothers tend to have boundary problems such as the bias and variance trade-off. There are many approaches to reducing the bias, but most of them do so at the cost of an increase in the variance of the estimator. Under smoothing will reduce the bias but will have a tendency of generating spurious peaks. Higher order smoothers can also be used but while this will lead to a smaller bias, the smoother will have a larger variance (Hengartner et al., 2009).

(Linton and Nielsen, 1994) developed a multiplicative technique for bias reduction and (Burr et al., 2010) have since used the approach in the smoothing of low resolution gamma spectra. The results obtained showed that the technique helped in the reduction of bias with no or negligible increase in variance. (Onsongo et al., 2018) also developed a nonparametric estimator for a finite population

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distribution function via simple random sampling without replacement with the aid this technique.

This paper considers estimation of a nonparametric estimator for finite population total and derivation of its asymptotic properties of a nonparametric distribution function estimator for a stratified population by utilizing the bias correction technique proposed by (Linton and Nielsen, 1994). The results obtained by (Linton and Nielsen, 1994) showed that the estimator of the regression function had desirable properties compared to existing estimators including solving the boundary problems hence the motivation to use it.

Outline of the paper

In section 2, we propose an estimator for finite population distribution function for a stratified population using a bias correction technique. In section 3, asymptotic properties of the estimator are derived. Empirical simulation of the results is given in section 4 and the conclusion of the findings is given in section 5.

2. Proposed Estimator

In this section, we develop a nonparametric estimator for a distribution function in the event of stratification of a finite population.

Consider a finite population of N units that can be classified into H strata each of size N_h where $h = 1, 2, \dots, H$ such that $N_1 + N_2 + \dots + N_H = N$.

Let $X_{hi}, i = 1, 2, \dots, N_h$ be the auxiliary variable for the h^{th} stratum with corresponding survey measurement $Y_{hi}, i = 1, 2, \dots, N_h$ from a common univariate distribution function.

Suppose that a simple random sample of size n_h is drawn without replacement from the h^{th} stratum such that the sample proportion $f_h = \frac{n_h}{N_h} \rightarrow 1$ as $n_h \rightarrow N_h$ and $N_h \rightarrow \infty$.

Then, the empirical distribution function for a finite population is defined as

$$F_N(t) = \frac{1}{N} \sum_{i=1}^N \Delta(t - y_i) \quad (1)$$

The corresponding estimator of a distribution function for a stratified population is defined as

$$\begin{aligned} \hat{F}_{Ny}^S(t) &= \frac{1}{N} \sum_{i=1}^H N_h \left(\frac{1}{N_h} \sum_{i=1}^{N_h} \Delta(t - y_{hi}) \right) \\ &= \frac{1}{N} \sum_{i=1}^H N_h F_{hy}(t) \end{aligned} \quad (2)$$

Where Δ denotes the step function of a given set, t is the α -quantile and i denotes the observation made from the h^{th} stratum.

$F_{hy}(t)$ is the h^{th} stratum distribution function for the random variable Y .

Let s be a sample of n_h units drawn from the h^{th} stratum via simple random sampling without replacement and $j \in r \in h = (N_h - s)$ be the non-sampled units in the h^{th} stratum.

Suppose the auxiliary information is known for all elements in the population while the survey variable is only observed for the sample elements.

Further, suppose that the survey variables are generated using a super population model defined by

$$Y_{hi} = \eta(x_{hi}) + \sigma(x_{hi})e_{hi} \quad (3)$$

Where e_{hi} 's are independent and identically distributed random variables with zero mean and variance $\sigma^2(x_{hi})$ with $E(Y_{hi}) = \eta(x_{hi})$ and

$$\text{Cov}(Y_{hi}, Y_{hj}) = \begin{cases} \sigma^2(x_{hi}) & \text{for } i = 1, 2, \dots, N_h \\ 0 & \text{otherwise} \end{cases}$$

Where $\eta(x_{hi})$ and $\sigma^2(x_{hi})$ are assumed to be smooth functions of x_{hi} .

The predictive form of the empirical distribution function for a stratified population under the model based approach therefore becomes

$$\hat{F}_{Ny}^S(t) = \frac{1}{N} \sum_{h=1}^H N_h \left(\frac{1}{N_h} \left[\sum_{i \in s \in h} \Delta(t - y_{hi}) + \sum_{j \in r \in h} \Delta(t - y_{hj}) \right] \right) \quad (4)$$

In this work, we propose the estimator for equation (4) as

$$\hat{F}_{MBC}^{(S)}(t) = \frac{1}{N} \sum_{h=1}^H N_h \left(\frac{1}{N_h} \left[\sum_{i \in s \in h} \Delta(t - y_{hi}) + \sum_{j \in r \in h} \hat{G}(t - \hat{\eta}_{n_h}(x_{hj})) \right] \right) \quad (5)$$

Where $\hat{\eta}_{n_h}(x_{hj})$ is the model-based nonparametric estimator for $\eta(x_{hj})$ and $\hat{G}(t - \hat{\eta}_{n_h}(x_{hj}))$ is the estimated distribution function of the residuals defined by $e_{hj} = y_{hj} - \hat{\eta}(x_{hj})$ using elements drawn from the h^{th} stratum.

Since $\sum_{i \in s \in h} \Delta(t - y_{hi})$ is known from the sample drawn, the task reduces to that of estimating $\sum_{j \in r \in h} \hat{G}(t - \hat{\eta}_{n_h}(x_{hj}))$.

To do this, the multiplicative bias correction technique is employed.

Suppose that $(X_{h1}, Y_{h1}), (X_{h2}, Y_{h2}), \dots, (X_{hN_h}, Y_{hN_h})$ are N independent pairs of random variables with the pair (X_{hi}, Y_{hi}) being real valued.

Define a pilot smoother of the regression function as

$$\tilde{\eta}_{n_h}(x_{hi}) = \sum_{i \in s \in h} w_i(x_{hi}; l) y_{hj} \quad (6)$$

Where $w_i(x_{hi}; l)$ are the Nadaraya-Watson kernel weights defined by

$$w_i(x_{hi}; l) = \frac{K_l(x - x_{hi})}{\sum_{i \in s \in h} K_l(x - x_{hi})} \text{ and } l \text{ is the bandwidth.}$$

$K(\cdot)$ is a function that is continuous, symmetric and bounded with real values.

Let the ratio $\lambda_{hi} = \frac{y_{hi}}{\tilde{\mu}_n(x_{hi})}$ be a noisy estimate of the inverse relative estimation error of the smoother $\tilde{\eta}_{n_h}$ given by $\frac{\eta(x)}{\tilde{\eta}_{n_h}(x_{hi})}$.

Smoothing λ_{hi} yields

$$\hat{\alpha}(x_{hi}) = \sum_{i=1}^{n_h} w_i(x_{hi}; l) \lambda_{hi} = \sum_{i=1}^{n_h} w_i(x_{hi}; l) \frac{y_i}{\tilde{\eta}_n(x_{hi})} \quad (7)$$

Equation (7) can then be used as a multiplicative correction of the pilot smoother in equation (6) which can now be defined by

$$\hat{\eta}_{n_h}(x_{hi}) = \hat{\alpha}(x_{hi})\tilde{\eta}_{n_h}(x_{hi}) \tag{8}$$

Assumptions

The following assumptions are made in the estimation of $\eta_{n_h}(x_{hi})$

1. The regression function is twice continuously differentiable everywhere.
2. The bandwidth l is such that $l \rightarrow 0, n_h l \rightarrow \infty$, as $n_h \rightarrow \infty$.

Using equation (7) in equation (8) yields

$$\hat{\eta}_{n_h}(x_{hi}) = \sum_{i=1}^{n_h} w_i(x_{hi}; l) \frac{\tilde{\eta}_{n_h}(x)}{\tilde{\eta}_{n_h}(x_{hi})} y_i \tag{9}$$

$$\hat{\eta}_{n_h}(x_{hi}) = \sum_{i=1}^{n_h} w_i(x_{hi}; l) \frac{\tilde{\eta}_{n_h}(x)}{\tilde{\eta}_{n_h}(x_{hi})} \eta(X_{hi}) +$$

$$\sum_{i=1}^{n_h} w_i(x_{hi}; l) \frac{\tilde{\eta}_{n_h}(x)}{\tilde{\eta}_{n_h}(x_{hi})} \{\varepsilon_{hi} + \eta(X_{hi})[b_{n_h}(x) - b_{n_h}(X_{hi})]\} +$$

$$\sum_{i=1}^{n_h} w_i(x_{hi}; l) \frac{\tilde{\eta}_{n_h}(x)}{\tilde{\eta}_{n_h}(x_{hi})} \varepsilon_{hi} [b_{n_h}(x) - b_{n_h}(X_{hi})] + O_p\left(\frac{1}{n_h l}\right) \tag{10}$$

For a detailed review on the derivation of $\hat{\eta}_{n_h}(x_{hi})$ see (Onsongo et al., 2018).

The estimator for the distribution function for a stratified population therefore becomes

$$\begin{aligned} \hat{F}_{MBC}^{(S)}(t) &= \frac{1}{N} \sum_{h=1}^H N_h \left(\frac{1}{N_h} \left[\sum_{i \in s \in h} \Delta(t - y_{hi}) + \right. \right. \\ &\sum_{j \in r \in h} \hat{G} \left(t - \left\{ \sum_{i=1}^{n_h} w_i(x_{hi}; l) \frac{\tilde{\eta}_{n_h}(x)}{\tilde{\eta}_{n_h}(X_{hi})} \eta(X_{hi}) + \right. \right. \\ &\sum_{i=1}^{n_h} w_i(x_{hi}; l) \frac{\tilde{\eta}_{n_h}(x)}{\tilde{\eta}_{n_h}(X_{hi})} \{\varepsilon_{hi} + \eta(X_{hi})[b_{n_h}(x) - \\ &b_{n_h}(X_{hi})]\} + \sum_{i=1}^{n_h} w_i(x_{hi}; l) \frac{\tilde{\eta}_{n_h}(x)}{\tilde{\eta}_{n_h}(X_{hi})} \varepsilon_{hi} [b_{n_h}(x) - \\ &b_{n_h}(X_{hi})] + O_p\left(\frac{1}{n_h l}\right) \left. \left. \right) \right) \end{aligned}$$

3. Asymptotic Properties of the Estimator under Stratified Sampling

3.1. Asymptotic Unbiasedness

Consider the asymptotic bias of the nonparametric estimator is defined as

$$\frac{1}{N} E[\hat{F}_{MBC}^{(S)}(t) - F_N(t)] \tag{10}$$

Where $\hat{F}_{MBC}^{(S)}(t) - F_N(t)$ is the estimated bias under stratified sampling.

Let $\hat{G}(t) = \sum_{i \in s \in h} w_{ij}^* I(y_{hi} - \hat{\eta}(x_{hj}))$ where $w_{ij}^* = \frac{w_{ij}}{\sum_{i \in s \in h} w_{ij}}$ where t is the α -quantile and w_{ij} are the weights that only take non-zero values for sample units $i \in s \in h$ with X_{hj} close to X_{hi} .

Equation (5) can then be written as

$$\hat{F}_{MBC}^{(S)}(t) = \frac{1}{N} \left\{ \sum_{i \in s \in h} \Delta(t - y_{hi}) + \sum_{j \in r \in h} \sum_{i \in s \in h} w_{ij}^* \Delta(t - \hat{e}_{hj}) \right\}$$

$$E[\hat{F}_{MBC}^{(S)}(t)] = \frac{1}{N} \left\{ \sum_{i \in s \in h} E[\Delta(t - y_{hi})] + \sum_{j \in r \in h} \sum_{i \in s \in h} w_{ij}^* E[\Delta(t - \hat{e}_{hj})] \right\}$$

However $\Delta(t - y_{hi}) \sim Bernoulli(P(y_{hi} \leq t))$ and $\sum_{i \in s \in h} \Delta(t - y_{hi}) \sim Binom(N_h P(y_{hi} \leq t))$ implying that

$$E[\hat{F}_{MBC}^{(S)}(t)] = \frac{n}{N} F_y(t) + \frac{F_e(t)}{N} \tag{12}$$

Next,

$$E[F_N(t)] = E\left[\frac{1}{N} \sum_{i=1}^N \Delta(t - y_i)\right] = F_y(t) \tag{13}$$

Substituting the results in equation (12) and equation (13) back to equation (10) yields

$$\begin{aligned} \frac{1}{N} E[\hat{F}_{MBC}^{(S)}(t) - F_N(t)] &= \frac{F_e(t)}{N^2} - \left(\frac{N-n}{N^2}\right) F_y(t) \\ &= \frac{1}{N} \left\{ \frac{F_e(t)}{N} - \left(1 - \frac{n}{N}\right) F_y(t) \right\} \end{aligned}$$

As $\frac{n}{N} \rightarrow 1$ and $N \rightarrow \infty, \left\{ \frac{F_e(t)}{N} - \left(1 - \frac{n}{N}\right) F_y(t) \right\} \rightarrow 0$

$\hat{F}_{MBC}^{(S)}(t)$ is therefore asymptotically unbiased.

3.2. Asymptotic Variance

Consider the estimated bias is given by

$$\begin{aligned} \hat{F}_{MBC}^{(S)}(t) - F_N(t) &= \frac{1}{N} \sum_{h=1}^H \left[\sum_{j \in r \in h} \sum_{i \in s \in h} w_{ij}^* \Delta(t - y_{hi}) - \frac{1}{N} \sum_{j \in r \in h} \Delta(t - y_{hj}) \right] \end{aligned}$$

The variance of the estimated bias can therefore be written as

$$\begin{aligned} Var[\hat{F}_{MBC}^{(S)}(t) - F_N(t)] &= \\ Var\left\{ \frac{1}{N} \sum_{h=1}^H \left[\sum_{j \in r \in h} \sum_{i \in s \in h} w_{ij}^* \Delta(t - y_{hi}) - \frac{1}{N} \sum_{j \in r \in h} \Delta(t - y_{hj}) \right] \right\} &= \\ Var[\hat{F}_{MBC}^{(S)}(t) - F_N(t)] &= \\ \frac{1}{N^2} \left\{ Var\left(\sum_{j \in r \in h} \sum_{i \in s \in h} w_{ij}^* \Delta(t - y_{hi})\right) + Var\left(\sum_{j \in r \in h} \Delta(t - y_{hj})\right) \right\} &\tag{14} \end{aligned}$$

The errors are assumed to be independent and identically distributed and therefore have zero covariance.

Consider $Var(\sum_{j \in r \in h} \sum_{i \in s \in h} w_{ij}^* \Delta(t - y_{hi}))$ and let $\hat{\varphi}_i(t) = \sum_{i \in s \in h} w_i^* \Delta(t - y_{hi})$

Then

$$Var(\sum_{j \in r \in h} \hat{\varphi}_i(t)) = \sum_{j \in r \in h} \sum_{k \in r \in h} Cov(\hat{\varphi}_j(t), \hat{\varphi}_k(t)) \tag{15}$$

With $\hat{\varphi}_i(t) = \sum_{i \in s \in h} w_i^* \Delta(t - y_{hi})$

Define $D_i(u) = P[\Delta(u - e_{hi})]$

$$\begin{aligned} Cov(\hat{\varphi}_j(t), \hat{\varphi}_k(t)) &= \\ = \sum_{i \in s \in h} w_{ij}^* w_{ik}^* \begin{bmatrix} D_i(t - \max(\hat{\eta}_{hj}, \hat{\eta}_{hk})) \\ -D_i(t - \hat{\eta}_{hj}) D_i(t - \hat{\eta}_{hk}) \end{bmatrix} &\tag{16} \end{aligned}$$

Suppose that $\hat{\eta}_{hj} < \hat{\eta}_{hk}$ whenever $j < k$ and suppose that the non-sampled units are labelled from 1 to $N_h - n_h$.

Then

$$\begin{aligned} &Var(\sum_{j \in r \in h} \sum_{i \in s \in h} w_{ij}^* \Delta(t - y_{hi})) \\ &= \sum_{i \in s \in h} \left\{ \sum_{j=1}^{N_h - n_h} \sum_{k=1}^{N_h - n_h} w_{ij}^* w_{ik}^* \left[\begin{array}{l} D_i(t - \max(\hat{\eta}_{hj}, \hat{\eta}_{hk})) \\ -D_i(t - \hat{\eta}_{hj})D_i(t - \hat{\eta}_{hk}) \end{array} \right] \right\} \end{aligned} \tag{17}$$

Next,

$$\begin{aligned} &Var(\sum_{j \in r \in h} \Delta(t - y_{hi})) = \sum_{j \in r \in h} Var[\Delta(t - y_{hi})] \\ &= (N_h - n_h)P(y_{hj} \leq t)[1 - P(y_{hj} \leq t)] \end{aligned} \tag{18}$$

Substituting equations (17) and (18) into equation (14) yields

$$\begin{aligned} &Var[\hat{F}_{MBC}^{(S)}(t) - F_N(t)] = \\ &\frac{1}{N^2} \left[\sum_{i \in s} \left\{ \sum_{j=1}^{N_h - n_h} \sum_{k=1}^{N_h - n_h} w_{ij}^* w_{ik}^* \left[\begin{array}{l} D_i(t - \max(\hat{\eta}_{hj}, \hat{\eta}_{hk})) \\ -D_i(t - \hat{\eta}_{hj})D_i(t - \hat{\eta}_{hk}) \end{array} \right] \right\} \right. \\ &\quad \left. + (N_h - n_h)P(y_{hj} \leq t)[1 - P(y_{hj} \leq t)] \right] \end{aligned} \tag{19}$$

It is clear from this result that

$$\lim_{N \rightarrow \infty} Var[\hat{F}_{MBC}^{(S)}(t) - F_N(t)] \rightarrow 0.$$

4. Results

In this section, simulation experiments were done to study the performance of the multiplicative bias corrected estimator for a stratified population.

Four populations of size 500 each are generated as $N(60, 4)$, $N(75, 4)$, $N(40, 3)$, $N(55, 5)$ such that there is a total of 2, 000 auxiliary variables.

The corresponding survey values y_i are generated using the super-population model

$y_i = \eta(x_i) + \varepsilon_i$ after which they are stratified according to form four strata. Proportional allocation was used to draw samples of size 100 from each stratum.

The estimators $\hat{F}_{Ny}^{rob}(t) = \frac{1}{N} \left[\sum_{i \in s} d_{it} + \sum_{j \in r} \hat{G}(t - b_v z_j) \right]$ suggested by (Chambers and Clark, 2012) and $\hat{F}_{RKM}(t) = \frac{1}{n} \sum_{i \in s} I(y_i \leq t) + \frac{1}{N} \sum_{j \in r} \hat{G}(t - \hat{a} - \hat{b}x_j) - \left(\frac{1}{n} - \frac{1}{N}\right) \sum_{i \in s} \hat{G}(t - \hat{a} - \hat{b}x_j)$ suggested by (Rao et al 1990) were then used in the comparison of results.

Table 1 shows the unconditional Relative Mean Error (RME) and Relative Root Mean Error (RRME) for the estimators at various values of the quantile α (i.e. 0.25, 0.5 and 0.75). Linear, quadratic and cosine mean functions were used to obtain the tabulated results. Similar results and conclusions can be obtained using other mean functions such as sine, cycle and bump.

The conditional Relative Mean Error and Relative Root Mean Error for an estimator $\tilde{F}_N(t)$ are calculated as:

$$\begin{aligned} RME &= \frac{1}{F_N(t)} \left\{ \frac{1}{100} \sum_{r=1}^{100} [\tilde{F}_{Ny}^m(t) - F_N(t)] \right\} \quad \text{and} \\ RRME &= \frac{1}{F_N(t)} \sqrt{\frac{1}{100} \sum_{r=1}^{100} [\tilde{F}_{Ny}^m(t) - F_N(t)]^2} \end{aligned}$$

respectively where m represents the level of iteration.

Table 2 shows the conditional Relative Mean Error (RME) and Relative Root Mean Error (RRME) for the estimators at various values of the quantile α (i.e. 0.25, 0.5 and 0.75).

Comparing the results in Table 1 and Table 2, it can be seen that $\hat{F}_{MBC}^{(S)}(t)$ has minimum Relative Mean Error and Relative Root Mean Error followed by $\hat{F}_{Ny}^{rob}(t)$ $\hat{F}_{RKM}(t)$ and at all levels of the α -quantile.

Table 1. Unconditional Relative Mean Errors and Relative Root Mean Errors

Mean Function	$\alpha = 0.25$					
	$\hat{F}_{MBC}^{(S)}(t)$		$\hat{F}_{Ny}^{rob}(t)$		$\hat{F}_{RKM}(t)$	
	RME	RRMSE	RME	RRMSE	RME	RRMSE
Linear	$1.165734 * 10^{-3}$	$1.93644 * 10^{-4}$	0.24277	0.07176	0.00352	0.06551
Quadratic	$-1.24553 * 10^{-4}$	$2.32966 * 10^{-6}$	0.02636	0.01422	0.03381	0.00282
Cosine	$6.21031 * 10^{-3}$	$2.03803 * 10^{-3}$	0.33354	0.01809	-0.03358	0.02466
$\alpha = 0.5$						
	RME	RRMSE	RME	RRMSE	RME	RRMSE
Linear	$4.85723 * 10^{-2}$	$2.70539 * 10^{-8}$	0.83112	0.03816	0.02742	0.03904
Quadratic	$6.93889 * 10^{-6}$	$2.07065 * 10^{-5}$	0.33992	0.23434	0.02154	0.01949
Cosine	$-9.992 * 10^{-4}$	$2.08525 * 10^{-4}$	0.17044	0.12143	0.01109	0.07599
$\alpha = 0.75$						
	RME	RRMSE	RME	RRMSE	RME	RRMSE
Linear	$7.5287 * 10^{-8}$	$1.75111 * 10^{-8}$	0.16864	0.15089	-0.04047	0.03535
Quadratic	$4.92662 * 10^{-5}$	$2.60271 * 10^{-8}$	0.17755	0.15877	0.00482	0.03087
Cosine	$2.5326 * 10^{-8}$	$1.86036 * 10^{-5}$	0.05416	0.00389	0.00111	0.00041

Table 2. Conditional Relative Mean Errors and Relative Root Mean Errors

Mean Function	$\alpha = 0.25$					
	$\hat{F}_{MBC}^{(S)}(t)$		$\hat{F}_{Ny}^{rob}(t)$		$\hat{F}_{RKM}(t)$	
	RME	RRMSE	RME	RRMSE	RME	RRMSE
Linear	0.00968	0.000758	0.01291	0.04046	0.82558	0.05287
Quadratic	0.00862	0.000274	0.09595	0.02788	0.46808	0.08229
Cosine	-0.00716	0.002085	0.03846	0.03292	0.01782	0.02193
$\alpha = 0.5$						
	RME	RRMSE	RME	RRMSE	RME	RRMSE
Linear	0.00621	0.000698	0.03189	0.02157	0.12696	0.10506
Quadratic	0.00307	0.001535	0.03295	0.23262	0.53046	0.04119
Cosine	-0.00208	0.012993	0.008253	0.01639	0.00545	0.00343
$\alpha = 0.75$						
	RME	RRMSE	RME	RRMSE	RME	RRMSE
Linear	0.00523	0.00117	0.01704	0.01513	0.07499	0.00363
Quadratic	0.00348	0.00161	0.01554	0.01393	0.05117	0.00301
Cosine	0.00237	0.01041	0.00259	0.00586	0.06373	0.08197

5. Conclusions

Use of $\hat{F}_{MBC}^{(S)}(t)$ has proved to yield results with great precision. $\hat{F}_{MBC}^{(S)}(t)$ can therefore be used in estimating distribution functions for stratified populations in various sectors.

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